

ALLAMA IQBAL OPEN UNIVERSITY

Department of Mathematics & Statistics

Course Code:	1429
Course Title:	Business Mathematics
Assignment Number:	01
Semester:	Autumn 2026
Academic Year:	2026

IMPORTANT NOTE

Assignments Last Date: 20-02-2027

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ALLAMA IQBAL OPEN UNIVERSITY, ISLAMABAD

(Department of Mathematics & Statistics)

Course: Business Mathematics (1429) Semester: Autumn 2026

Level: Bachelors / AD Assignment No. 01

Q.1 Probability Theory (20 Marks)

- Find the sample space for choosing an odd number from 1 to 15 at random.
- What is the difference between mutually exclusive events and collectively exhaustive events?
- In a game show, the contestant is shown 10 boxes, 3 of which contain prizes. If a contestant is allowed to select any three boxes, what is the probability that:
 - The contestant wins all three prizes.
 - Only one selected box contains a prize.
- The probability that an applicant for pilot school will be admitted is 0.5. If three applicants are selected at random, what is the probability that:
 - All three will be admitted
 - None will be admitted
 - Only one will be admitted

Solution:

a) Sample Space for Odd Numbers from 1 to 15: The sample space S containing all odd integers from 1 to 15 inclusive:

$$S = \{1, 3, 5, 7, 9, 11, 13, 15\}$$

b) Mutually Exclusive vs. Collectively Exhaustive Events:

- Mutually Exclusive Events:** Two or more events that cannot occur simultaneously in a single trial. If A occurs, B cannot occur ($A \cap B = \emptyset \implies P(A \cap B) = 0$).
- Collectively Exhaustive Events:** A set of events whose union accounts for all possible outcomes in the sample space S ($\bigcup_{i=1}^n A_i = S \implies \sum P(A_i) = 1$).

c) Game Show Prize Box Probabilities: Total boxes $N = 10$, prize boxes = 3, non-prize boxes = 7. Total ways to select 3 boxes:

$$\binom{10}{3} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120$$

I. All 3 prizes: Selecting 3 prize boxes out of 3: $\binom{3}{3} = 1$.

$$P(\text{All 3 prizes}) = \frac{1}{120} \approx 0.00833 \quad (0.833\%)$$

II. Only 1 prize: Selecting 1 prize box out of 3 AND 2 non-prize boxes out of 7:

$$\binom{3}{1} \times \binom{7}{2} = 3 \times 21 = 63$$

$$P(\text{Only 1 prize}) = \frac{63}{120} = \frac{21}{40} = \mathbf{0.525} \quad (\mathbf{52.5\%})$$

d) Pilot School Admissions (Binomial Distribution, $n = 3, p = 0.5, q = 0.5$): Binomial Formula: $P(X = k) = \binom{n}{k} p^k q^{n-k}$

i. All three admitted ($X = 3$):

$$P(X = 3) = \binom{3}{3} (0.5)^3 (0.5)^0 = 1 \times 0.125 \times 1 = \mathbf{0.125} \quad \left(\frac{1}{8}\right)$$

ii. None admitted ($X = 0$):

$$P(X = 0) = \binom{3}{0} (0.5)^0 (0.5)^3 = 1 \times 1 \times 0.125 = \mathbf{0.125} \quad \left(\frac{1}{8}\right)$$

iii. Only one admitted ($X = 1$):

$$P(X = 1) = \binom{3}{1} (0.5)^1 (0.5)^2 = 3 \times 0.5 \times 0.25 = \mathbf{0.375} \quad \left(\frac{3}{8}\right)$$

Q.2 Random Variables (20 Marks)

- a) Let X be a random variable having a normal distribution with mean 48 and standard deviation 10. Then find $P(X \leq 41)$.
- b) Define a random variable. What is the difference between a discrete random variable and a continuous random variable?
- c) Construct the discrete probability distribution that corresponds to the experiment of tossing a fair coin three times. Suppose the random variable X equals the number of heads occurring in three tosses. What is the probability of two or more heads?
- d) Compute the mean, median, mode, and standard deviation for the following discrete probability distribution:
- | | | | | | |
|--------------------|----|----|----|----|----|
| Value of X: | 20 | 30 | 40 | 50 | 60 |
| Frequency: | 8 | 12 | 10 | 12 | 8 |

Solution:

- a) **Normal Distribution** $P(X \leq 41)$ for $\mu = 48, \sigma = 10$: Convert $X = 41$ to Z -score:

$$Z = \frac{X - \mu}{\sigma} = \frac{41 - 48}{10} = -0.70$$

Using standard normal distribution tables:

$$P(X \leq 41) = P(Z \leq -0.70) = 1 - \Phi(0.70) = 1 - 0.7580 = \mathbf{0.2420} \quad (\mathbf{24.20\%})$$

b) Random Variable & Types:

- **Random Variable:** A numerical description of the outcome of an experiment ($X : S \rightarrow \mathbb{R}$).
- **Discrete vs. Continuous:** A discrete random variable takes on countable isolated values (e.g., number of defective items), whereas a continuous random variable takes any value along a continuous range (e.g., time, weight, height).

- c) **Coin Tossing Probability Distribution** ($n = 3$ tosses): Sample space $S = \{HHH, HHT, HTH, HTT, THT, THT, TTH, TTT\}$, $|S| = 8$.

Number of Heads (X)	Outcomes	Probability $P(X)$
0	$\{TTT\}$	$1/8 = 0.125$
1	$\{HTT, THT, TTH\}$	$3/8 = 0.375$
2	$\{HHT, HTH, THH\}$	$3/8 = 0.375$
3	$\{HHH\}$	$1/8 = 0.125$

Probability of two or more heads ($X \geq 2$):

$$P(X \geq 2) = P(X = 2) + P(X = 3) = \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \mathbf{0.50} \quad (\mathbf{50\%})$$

d) Statistics for Frequency Distribution ($N = \sum f = 50$):

X	f	fX	$(X - \mu)$	$f(X - \mu)^2$
20	8	160	-20	3200
30	12	360	-10	1200
40	10	400	0	0
50	12	600	10	1200
60	8	480	20	3200
Total	50	2000	-	8800

1. **Mean (μ):** $\mu = \frac{\sum fX}{N} = \frac{2000}{50} = 40$

2. **Median:** $\frac{N}{2} = 25$. Cumulative frequency reaches 25 at $X = 40$. Thus, Median = **40**.

3. **Mode:** Maximum frequency 12 occurs at both $X = 30$ and $X = 50$. Thus, Modes = **30** and **50** (Bimodal).

4. **Standard Deviation (σ):** $\sigma = \sqrt{\frac{\sum f(X-\mu)^2}{N}} = \sqrt{\frac{8800}{50}} = \sqrt{176} \approx 13.266$

Q.3 Equations (20 Marks)

Solve the following first-degree equations:

a) $x - 56 = 50x + 30$

b) $-15 + 3x = 24x - 3$

c) $(4x + 96) - (-16 + 4x) + 45 = 0$

Solution:a) $x - 56 = 50x + 30$: Rearrange terms:

$$x - 50x = 30 + 56$$

$$-49x = 86$$

$$x = -\frac{86}{49} \approx -1.7551$$

b) $-15 + 3x = 24x - 3$: Rearrange terms:

$$3x - 24x = -3 + 15$$

$$-21x = 12$$

$$x = -\frac{12}{21} = -\frac{4}{7} \approx -0.5714$$

c) $(4x + 96) - (-16 + 4x) + 45 = 0$: Expand and remove parentheses:

$$4x + 96 + 16 - 4x + 45 = 0$$

$$(4x - 4x) + (96 + 16 + 45) = 0$$

$$0x + 157 = 0 \implies 157 = 0$$

Since $157 = 0$ is false, the equation is inconsistent. **Conclusion: No Solution.****Q.4 Quadratic Equations & Applied Geometry (20 Marks)**

a) Solve the following quadratic equations using the quadratic formula:

i. $5x^2 + 4x - 1 = 0$

ii. $4t^2 - 4 = 0$

b) A railing is to enclose a rectangular area of 1,600 square feet. The length of the plot is twice the width. How much railing must be used?

Solution:

a) Solve using Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$:

i. $5x^2 + 4x - 1 = 0$ ($a = 5, b = 4, c = -1$):

$$x = \frac{-4 \pm \sqrt{4^2 - 4(5)(-1)}}{2(5)} = \frac{-4 \pm \sqrt{16 + 20}}{10} = \frac{-4 \pm \sqrt{36}}{10} = \frac{-4 \pm 6}{10}$$

$$x_1 = \frac{-4 + 6}{10} = \frac{2}{10} = \mathbf{0.2} \quad \left(\frac{1}{5}\right), \quad x_2 = \frac{-4 - 6}{10} = \frac{-10}{10} = \mathbf{-1}$$

ii. $4t^2 - 4 = 0$ ($a = 4, b = 0, c = -4$):

$$4t^2 = 4 \implies t^2 = 1 \implies \mathbf{t = \pm 1}$$

b) Rectangular Plot Railing Calculation: Let width = W , length $L = 2W$, Area $A = 1600$ sq ft.

$$A = L \times W = (2W) \times W = 2W^2 = 1600$$

$$W^2 = 800 \implies W = \sqrt{800} = 20\sqrt{2} \approx 28.284 \text{ feet}$$

$$L = 2W = 40\sqrt{2} \approx 56.569 \text{ feet}$$

Perimeter (Total Railing Required):

$$P = 2(L + W) = 2(40\sqrt{2} + 20\sqrt{2}) = 2(60\sqrt{2}) = 120\sqrt{2} \approx \mathbf{169.71} \text{ feet}$$

Q.5 Linear Equations (20 Marks)

- a) Solve the linear equation $y = 9x + 81$.
- b) A company has fixed costs of \$9,000 for plant and equipment and variable costs of \$600 for each unit of output. What is the total cost at varying levels of output?
- c) Find the equation of the straight line that has slope $m = 3$ and passes through the point $(-4, -6)$.

Solution:

a) Solve $y = 9x + 81$:

- x -intercept (set $y = 0$): $0 = 9x + 81 \implies 9x = -81 \implies \mathbf{x = -9}$
- y -intercept (set $x = 0$): $y = 9(0) + 81 \implies \mathbf{y = 81}$
- Explicit expression for x : $x = \frac{y-81}{9}$

b) **Total Cost Model:** Fixed Cost $FC = \$9,000$, Variable Cost per unit $VC = \$600$. For q units of output:

$$\mathbf{TC(q) = 9000 + 600q}$$

c) **Equation of Line ($m = 3$, Point $(-4, -6)$):** Point-slope formula $y - y_1 = m(x - x_1)$:

$$y - (-6) = 3(x - (-4))$$

$$y + 6 = 3(x + 4)$$

$$y + 6 = 3x + 12 \implies \mathbf{y = 3x + 6}$$