Allama Iqbal Open University

Solved Assignments Spring 2026

Course Code:	**1429 Code**
Course Name:	**Business Mathematics**
Class:	**BA/BCom/AD**
Total Credit Hours	**3**
Total Assignments	**2**

Assignment 2

Q.1 Matrices

a) Solve the following system of linear equations using matrices.

$$
\begin{aligned}
2x_1 + 3x_2 - 4x_3 &= -4 \\
x_1 - x_2 + x_3 &= 2 \\
x_1 + 2x_2 + 3x_3 &= 14
\end{aligned}
$$

Solution:

Write the system in matrix form $AX = B$:

$$A = \begin{bmatrix} 2 & 3 & -4 \\ 1 & -1 & 1 \\ 1 & 2 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad B = \begin{bmatrix} -4 \\ 2 \\ 14 \end{bmatrix}$$

We need to find $X = A^{-1}B$.

Find determinant of A

$$\det(A) = 2\begin{vmatrix} -1 & 1 \\ 2 & 3 \end{vmatrix} - 3\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} + (-4)\begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix}$$

$$= 2[(-1)(3) - (1)(2)] - 3[(1)(3) - (1)(1)] - 4[(1)(2) - (-1)(1)]$$

$$= 2[-3 - 2] - 3[3 - 1] - 4[2 + 1]$$

$$= 2(-5) - 3(2) - 4(3) = -10 - 6 - 12 = -28$$

Since $\det(A) \neq 0$, the inverse exists.

Find matrix of cofactors

$$C_{11} = +\begin{vmatrix} -1 & 1 \\ 2 & 3 \end{vmatrix} = (-1)(3) - (1)(2) = -3 - 2 = -5$$

$$C_{12} = -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -[(1)(3) - (1)(1)] = -[3 - 1] = -2$$

$$C_{13} = +\begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} = (1)(2) - (-1)(1) = 2 + 1 = 3$$

$$C_{21} = -\begin{vmatrix} 3 & -4 \\ 2 & 3 \end{vmatrix} = -[(3)(3) - (-4)(2)] = -[9 + 8] = -17$$

$$C_{22} = +\begin{vmatrix} 2 & -4 \\ 1 & 3 \end{vmatrix} = (2)(3) - (-4)(1) = 6 + 4 = 10$$

$$C_{23} = -\begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = -[(2)(2) - (3)(1)] = -[4 - 3] = -1$$

$$C_{31} = +\begin{vmatrix} 3 & -4 \\ -1 & 1 \end{vmatrix} = (3)(1) - (-4)(-1) = 3 - 4 = -1$$

$$C_{32} = -\begin{vmatrix} 2 & -4 \\ 1 & 1 \end{vmatrix} = -[(2)(1) - (-4)(1)] = -[2 + 4] = -6$$

$$C_{33} = +\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = (2)(-1) - (3)(1) = -2 - 3 = -5$$

Cofactor matrix:

$$\text{Cof}(A) = \begin{bmatrix} -5 & -2 & 3 \\ -17 & 10 & -1 \\ -1 & -6 & -5 \end{bmatrix}$$

$$\text{adj}(A) = \begin{bmatrix} -5 & -17 & -1 \\ -2 & 10 & -6 \\ 3 & -1 & -5 \end{bmatrix}$$

Inverse matrix

$$A^{-1} = \frac{1}{\det(A)} \cdot \text{adj}(A) = \frac{1}{-28} \begin{bmatrix} -5 & -17 & -1 \\ -2 & 10 & -6 \\ 3 & -1 & -5 \end{bmatrix}$$

Find $X = A^{-1}B$

$$X = \frac{1}{-28} \begin{bmatrix} -5 & -17 & -1 \\ -2 & 10 & -6 \\ 3 & -1 & -5 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 14 \end{bmatrix}$$

$$= \frac{1}{-28} \begin{bmatrix} (-5)(-4) + (-17)(2) + (-1)(14) \\ (-2)(-4) + (10)(2) + (-6)(14) \\ (3)(-4) + (-1)(2) + (-5)(14) \end{bmatrix}$$

$$= \frac{1}{-28} \begin{bmatrix} 20 - 34 - 14 \\ 8 + 20 - 84 \\ -12 - 2 - 70 \end{bmatrix}$$

$$= \frac{1}{-28} \begin{bmatrix} -28 \\ -56 \\ -84 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-28}{-28} \\ \frac{-56}{-28} \\ \frac{-84}{-28} \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\boxed{x_1 = 1,\ x_2 = 2,\ x_3 = 3}$$

Q.1 Matrices

b) Given

$$A = \begin{bmatrix} 2 & -4 & 1 \\ 4 & 0 & -3 \\ -1 & 3 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 0 \\ 1 & 2 \\ 2 & 1 \end{bmatrix}$$

find the product AB.

b) Given

$$A = \begin{bmatrix} 2 & -4 & 1 \\ 4 & 0 & -3 \\ -1 & 3 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 0 \\ 1 & 2 \\ 2 & 1 \end{bmatrix}$$

find the product AB.

Solution:

$$AB = \begin{bmatrix} 2 & -4 & 1 \\ 4 & 0 & -3 \\ -1 & 3 & -2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (2)(3)+(-4)(1)+(1)(2) & (2)(0)+(-4)(2)+(1)(1) \\ (4)(3)+(0)(1)+(-3)(2) & (4)(0)+(0)(2)+(-3)(1) \\ (-1)(3)+(3)(1)+(-2)(2) & (-1)(0)+(3)(2)+(-2)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 6-4+2 & 0-8+1 \\ 12+0-6 & 0+0-3 \\ -3+3-4 & 0+6-2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -7 \\ 6 & -3 \\ -4 & 4 \end{bmatrix}$$

$$\boxed{AB = \begin{bmatrix} 4 & -7 \\ 6 & -3 \\ -4 & 4 \end{bmatrix}}$$

c) Mr. Asad bought 3 kg of sugar, 10 kg of wheat and 1 kg of salt from the local market at the rates of Rs. 50, Rs. 20 and Rs. 15 per kg, respectively. If the prices of these items on the Sunday market are Rs. 45, Rs. 16 and Rs. 12 per kg respectively and the one-way fare to the Sunday market is Rs. 15. Using matrices multiplication, find his net profit or loss.

Solution:

Let the quantity matrix (row vector) be:

$$Q = \begin{bmatrix} 3 & 10 & 1 \end{bmatrix}$$

Local market price matrix (column vector):

$$P_{\text{local}} = \begin{bmatrix} 50 \\ 20 \\ 15 \end{bmatrix}$$

Sunday market price matrix (column vector):

$$P_{\text{sunday}} = \begin{bmatrix} 45 \\ 16 \\ 12 \end{bmatrix}$$

Total cost at local market:

$$\text{Cost}_{\text{local}} = Q \times P_{\text{local}} = \begin{bmatrix} 3 & 10 & 1 \end{bmatrix} \begin{bmatrix} 50 \\ 20 \\ 15 \end{bmatrix}$$

$$= (3)(50) + (10)(20) + (1)(15) = 150 + 200 + 15 = 365$$

Total cost at Sunday market (excluding fare):

$$\text{Cost}_{\text{sunday}} = Q \times P_{\text{sunday}} = \begin{bmatrix} 3 & 10 & 1 \end{bmatrix} \begin{bmatrix} 45 \\ 16 \\ 12 \end{bmatrix}$$

$$= (3)(45) + (10)(16) + (1)(12) = 135 + 160 + 12 = 307$$

Total cost at Sunday market including one-way fare:

$$\text{Total Sunday cost} = 307 + 15 = 322$$

Net profit (or loss) if he goes to Sunday market instead of local:

$$\text{Profit} = \text{Cost}_{\text{local}} - \text{Total Sunday cost} = 365 - 322 = 43$$

Since the result is positive, it is a **net profit** of Rs. 43.

$$\boxed{\text{Net profit} = \text{Rs. } 43}$$

Q.2 Determinants and Inverses of Matrices

a) Evaluate the following determinants:

i.

$$\begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 0 \\ 2 & -3 & 5 \end{vmatrix}$$

Solution:

Expand along the first row:

$$= 2\begin{vmatrix} 1 & 0 \\ -3 & 5 \end{vmatrix} - 3\begin{vmatrix} 1 & 0 \\ 2 & 5 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix}$$

$$= 2[(1)(5) - (0)(-3)] - 3[(1)(5) - (0)(2)] - 1[(1)(-3) - (1)(2)]$$

$$= 2[5 - 0] - 3[5 - 0] - 1[-3 - 2]$$

$$= 2(5) - 3(5) - 1(-5) = 10 - 15 + 5 = 0$$

$$\boxed{0}$$

ii.

$$\begin{vmatrix} 2a & a & a \\ b & 2b & b \\ c & c & 2c \end{vmatrix}$$

Solution:

$$= 2a\begin{vmatrix} 2b & b \\ c & 2c \end{vmatrix} - a\begin{vmatrix} b & b \\ c & 2c \end{vmatrix} + a\begin{vmatrix} b & 2b \\ c & c \end{vmatrix}$$

$$= 2a[(2b)(2c) - (b)(c)] - a[(b)(2c) - (b)(c)] + a[(b)(c) - (2b)(c)]$$

$$= 2a[4bc - bc] - a[2bc - bc] + a[bc - 2bc]$$

$$= 2a(3bc) - a(bc) + a(-bc) = 6abc - abc - abc = 4abc$$

c) If $A = \begin{bmatrix} 1 & -2 & -3 \\ 2 & 0 & 1 \\ -4 & 6 & 8 \end{bmatrix}$**, find** $|A|$**.**

Solution:

$$|A| = \begin{vmatrix} 1 & -2 & -3 \\ 2 & 0 & 1 \\ -4 & 6 & 8 \end{vmatrix}$$

$$= 1\begin{vmatrix} 0 & 1 \\ 6 & 8 \end{vmatrix} - (-2)\begin{vmatrix} 2 & 1 \\ -4 & 8 \end{vmatrix} + (-3)\begin{vmatrix} 2 & 0 \\ -4 & 6 \end{vmatrix}$$

$$= 1[(0)(8) - (1)(6)] + 2[(2)(8) - (1)(-4)] - 3[(2)(6) - (0)(-4)]$$

$$= 1(0-6) + 2(16+4) - 3(12-0) = -6 + 2(20) - 36 = -6 + 40 - 36 = -2$$

$$\boxed{-2}$$

Q.3 Differentiation

a) Find the derivative of the function $f(x) = 15x^{100} - 3x^{12} + 5x - 46$.

Solution:

$$f'(x) = \frac{d}{dx}(15x^{100}) - \frac{d}{dx}(3x^{12}) + \frac{d}{dx}(5x) - \frac{d}{dx}(46)$$

$$= 15 \cdot 100x^{99} - 3 \cdot 12x^{11} + 5 \cdot 1 - 0$$

$$= 1500x^{99} - 36x^{11} + 5$$

$$\boxed{f'(x) = 1500x^{99} - 36x^{11} + 5}$$

b) For the function $f(x) = x^2 - 2x + 3$ find the point where the tangent line of this function is horizontal.

Solution:

The tangent line is horizontal where the derivative equals zero.

$$f'(x) = 2x - 2$$

Set $f'(x) = 0$:

$$2x - 2 = 0 \implies 2x = 2 \implies x = 1$$

Find the corresponding y value:

$$f(1) = (1)^2 - 2(1) + 3 = 1 - 2 + 3 = 2$$

$$\boxed{(1, 2)}$$

c) The distance covered by a car in kilometers is given as a function of time in hours by the following relation $f(t) = t^2 + 50t + 15$. Find the instantaneous velocity of the car at $t = 2$.

Solution:

Instantaneous velocity is the derivative of distance with respect to time.

$$f'(t) = 2t + 50$$

At $t = 2$:

$$f'(2) = 2(2) + 50 = 4 + 50 = 54$$

$$\boxed{54 \text{ km/h}}$$

d) The position of a particle moving along a straight line at time t is given by $s = f(t) = 2t^2 + 7$. Find the instantaneous rate of change at $t = 12$ seconds.

Solution:

Instantaneous rate of change is the derivative.

$$f'(t) = 4t$$

At $t = 12$:

$$f'(12) = 4(12) = 48$$

$$\boxed{48 \text{ units per second}}$$

e) The production costs per week for producing x widgets in a factory is given by $C(x) = 500 + 350x - 0.09x^2$. Calculate the cost to produce the 301st widget at $x = 300$.

Solution:

The cost of the 301st widget is the marginal cost at $x = 300$, i.e., $C'(300)$.

$$C'(x) = 350 - 0.18x$$

At $x = 300$:

$$C'(300) = 350 - 0.18(300) = 350 - 54 = 296$$

$$\boxed{296 \text{ dollars}}$$

Q.4 Partial Derivatives

a) Find the 2nd order derivative of the function: $f(x) = \sqrt[3]{x^5}$

Solution:

$$f(x) = x^{5/3}$$

$$\frac{d}{dx}f(x) = \frac{5}{3}x^{2/3}$$

$$\frac{d^2}{dx^2}f(x) = \frac{5}{3}\cdot\frac{2}{3}x^{-1/3} = \frac{10}{9}x^{-1/3}$$

$$\boxed{f''(x) = \frac{10}{9\sqrt[3]{x}}}$$

b) Find the values of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point $(4, -5)$ if $f(x, y) = x^2 + 3xy^2 + y - 1$.

Solution:

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial x}(3xy^2) + \frac{\partial}{\partial x}(y) - \frac{\partial}{\partial x}(1) = 2x + 3y^2 + 0 - 0$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2) + \frac{\partial}{\partial y}(3xy^2) + \frac{\partial}{\partial y}(y) - \frac{\partial}{\partial y}(1) = 0 + 6xy + 1 - 0$$

At $(4, -5)$:

$$\frac{\partial f}{\partial x}(4, -5) = 2(4) + 3(-5)^2 = 8 + 75 = 83$$

$$\frac{\partial f}{\partial y}(4, -5) = 6(4)(-5) + 1 = -120 + 1 = -119$$

$$\boxed{\frac{\partial f}{\partial x} = 83, \quad \frac{\partial f}{\partial y} = -119}$$

c) Find the second partial derivatives of the function $f(x, y) = 3x^2y + 2y^3$.

Solution:

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(3x^2y) + \frac{\partial}{\partial x}(2y^3) = 6xy + 0 = 6xy$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(3x^2y) + \frac{\partial}{\partial y}(2y^3) = 3x^2 + 6y^2$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}(6xy) = 6y$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}(3x^2 + 6y^2) = 12y$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial y}(6xy) = 6x$$

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial x}(3x^2 + 6y^2) = 6x$$

$$\boxed{f_{xx} = 6y, \; f_{yy} = 12y, \; f_{xy} = f_{yx} = 6x}$$

d) If $f(x, y) = x^2 + y^2 + 2xy$, find the critical points and determine if they are maxima or minima.

Solution:

$$\frac{\partial f}{\partial x} = 2x + 2y$$

$$\frac{\partial f}{\partial y} = 2y + 2x$$

Set $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$:

$$2x + 2y = 0 \implies x + y = 0 \implies y = -x$$

Both equations give the same condition. Critical points: all points on the line $y = -x$.

Second partial derivatives:

$$\frac{\partial^2 f}{\partial x^2} = 2, \quad \frac{\partial^2 f}{\partial y^2} = 2, \quad \frac{\partial^2 f}{\partial x \partial y} = 2$$

$$D = f_{xx}f_{yy} - (f_{xy})^2$$

Second partial derivatives:

$$\frac{\partial^2 f}{\partial x^2} = 2, \quad \frac{\partial^2 f}{\partial y^2} = 2, \quad \frac{\partial^2 f}{\partial x \partial y} = 2$$

$$D = \frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 = (2)(2) - (2)^2 = 4 - 4 = 0$$

Test inconclusive. Observe $f(x, y) = (x + y)^2 \geq 0$ and $f = 0$ when $x + y = 0$. Hence all critical points are minima with minimum value 0.

$$\boxed{\text{Critical points: } y = -x \text{ (all minima, value 0)}}$$

Q.5 Optimization

a) Find the interval on which f is increasing, decreasing, concave up, and concave down for $f(x) = 2x^3 - 4x - 1$.

Solution:

$$f(x) = 2x^3 - 4x - 1$$

First Derivative:

$$f'(x) = 6x^2 - 4$$

Critical Points (Stationary Points):

Set $f'(x) = 0$:

$$6x^2 - 4 = 0 \implies x^2 = \frac{2}{3} \implies x = \pm\sqrt{\frac{2}{3}} \approx \pm 0.8165$$

Interval	Test Value	$f'(x) = 6x^2 - 4$	Sign	Increasing / Decreasing
$(-\infty, -0.8165)$	$x = -1$	$6(1) - 4 = 2$	$+$	Increasing
$(-0.8165, 0.8165)$	$x = 0$	$0 - 4 = -4$	$-$	Decreasing
$(0.8165, \infty)$	$x = 1$	$6 - 4 = 2$	$+$	Increasing

Second Derivative:

$$f''(x) = 12x$$

Inflection Point: Set $f''(x) = 0$:

$$12x = 0 \implies x = 0$$

Concavity Table:

Interval	Test Value	$f''(x) = 12x$	Sign	Concave Up / Down
$(-\infty, 0)$	$x = -1$	-12	$-$	Concave Down
$(0, \infty)$	$x = 1$	12	$+$	Concave Up

Stationary Values:

At $x \approx -0.8165$:

$$f(-0.8165) = 2(-0.8165)^3 - 4(-0.8165) - 1 = 2(-0.5443) + 3.266 - 1$$
$$= -1.0886 + 3.266 - 1 = 1.1774$$

At $x \approx 0.8165$:

$$f(0.8165) = 2(0.5443) - 4(0.8165) - 1 = 1.0886 - 3.266 - 1 = -3.1774$$

$$\boxed{\text{Increasing: } (-\infty, -0.8165) \cup (0.8165, \infty)}$$

$$\boxed{\text{Decreasing: } (-0.8165, 0.8165)}$$

$$\boxed{\text{Concave up: } (0, \infty)}$$

$$\boxed{\text{Concave down: } (-\infty, 0)}$$

$$\boxed{\text{Local maximum at } x \approx -0.8165,\ f \approx 1.1774}$$

$$\boxed{\text{Local minimum at } x \approx 0.8165,\ f \approx -3.1774}$$

b) A company manufactures two types of a certain product. The joint cost function for producing x units of product A and y units of product B is given by $c = x^2 + 8xy + 40$. Find the quantity of each that results in the lowest cost.

Solution:

$$c(x, y) = x^2 + 8xy + 40$$

Find the first partial derivatives:

$$\frac{\partial c}{\partial x} = 2x + 8y$$

$$\frac{\partial c}{\partial y} = 8x$$

Find critical points:

Set both partial derivatives equal to zero:

$$2x + 8y = 0 \quad \text{and} \quad 8x = 0$$

From $8x = 0$, we get $x = 0$.

Substitute $x = 0$ into $2x + 8y = 0$:

$$2(0) + 8y = 0 \implies 8y = 0 \implies y = 0$$

Thus, the only critical point is $(0, 0)$.

Second partial derivatives:

$$\frac{\partial^2 c}{\partial x^2} = 2, \quad \frac{\partial^2 c}{\partial y^2} = 0, \quad \frac{\partial^2 c}{\partial x \partial y} = 8$$

Discriminant:

$$D = \frac{\partial^2 c}{\partial x^2} \cdot \frac{\partial^2 c}{\partial y^2} - \left(\frac{\partial^2 c}{\partial x \partial y} \right)^2 = (2)(0) - (8)^2 = -64 < 0$$

$$\boxed{\text{Minimum cost} = 40}$$

c) A manufacturer has determined a cost function which expresses the annual cost of purchasing, owing and maintaining its raw material inventory (C) as a function of the size of each order (q). The cost function is

$$C = \frac{51200}{q} + 80q + 750000$$

i. Determine the order size q which minimizes inventory cost C.

ii. What are the minimum inventory costs expected to equal?

Solution:

$$C(q) = \frac{51200}{q} + 80q + 750000, \quad q > 0$$

First derivative:

$$C'(q) = -\frac{51200}{q^2} + 80$$

Set $C'(q) = 0$:

$$-\frac{51200}{q^2} + 80 = 0 \implies \frac{51200}{q^2} = 80 \implies q^2 = \frac{51200}{80} = 640$$

$$q = \sqrt{640} \approx 25.2982$$

Second derivative (to confirm minimum):

$$C''(q) = \frac{102400}{q^3} > 0 \quad \text{for all } q > 0$$

Thus, $q \approx 25.30$ gives a minimum.

Minimum cost:

$$C(25.2982) = \frac{51200}{25.2982} + 80(25.2982) + 750000$$

$$\frac{51200}{25.2982} \approx 2023.86$$

$$80 \times 25.2982 \approx 2023.86$$

$$C \approx 2023.86 + 2023.86 + 750000 = 754047.72$$

d) The total cost and total revenue functions for a product are

$$C(q) = 500 + 100q + 0.5q^2 \quad \text{and} \quad R(q) = 500q.$$

Find the profit-maximizing level of output.

Solution:

Profit function $P(q)$ = Revenue – Cost

$$P(q) = R(q) - C(q) = 500q - (500 + 100q + 0.5q^2)$$

$$P(q) = 500q - 500 - 100q - 0.5q^2$$

$$P(q) = 400q - 0.5q^2 - 500$$

First derivative:

$$P'(q) = \frac{d}{dq}(400q) - \frac{d}{dq}(0.5q^2) - \frac{d}{dq}(500) = 400 - q$$

Set $P'(q) = 0$ to find critical point:

$$400 - q = 0 \implies q = 400$$

Since $P''(q) = -1 < 0$ for all q, the function is concave down and the critical point gives a **maximum.**

Maximum profit:

$$P(400) = 400(400) - 0.5(400)^2 - 500$$

$$= 160000 - 0.5(160000) - 500 = 160000 - 80000 - 500 = 79500$$

Answer:

$$\boxed{q = 400 \text{ units}}$$